Correlating PSI and CUP

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Having inherited the curiosity gene, I just can't resist fiddling with things. And one of the things I can't resist fiddling with is firearms. I think I am the only kid in town that asked for, and got, a Fabrique Scientific strain gauge system for Christmas, and promptly stuck it on his trusty 30-06. So I suppose that it is only natural that I'd be curious about how CUP and PSI work. That's what this article is about.

History

The Lyman reloading manual is one of my favorites. It's clearly written, a pleasure to read, and it sheds some interesting light on the history of terminology in the measurement of chamber pressure. Before about the 1960's the only measurement system we had for chamber pressure was the copper crusher method. Up until that time, what we now call CUP was commonly known by two different names: CUP and PSI. The two were used practically interchangeably. Of course, this use of PSI was incorrect. It wasn't much of a problem until piezoelectric and strain gauge systems became commonly available. These systems, of course really do measure in PSI. When they arrived on the scene, it caused a lot of concern and confusion. "For years, 52,000 PSI (crusher method with erroneous designation) had been published as maximum for the 270 Win. Suddenly, there were new publications showing 65,000 PSI …as maximum."[1]

If you look at any publications before about 1965, and they say that PSI and CUP are not the same, and that you should not attempt to convert one to the other, they are talking about the old, incorrect use of the term PSI, not the modern, correct use of PSI from strain gauges and piezoelectric pressure meters.

What is Correlation?

If you're on one of the reloading bulletin boards, and say that PSI (modern use) and CUP are correlated, you'd best be wearing your asbestos underwear. There are a lot of people that "know" that the two systems aren't correlated, and will tell you so in no uncertain terms. Math and physics aren't on their side, as we shall see. I suspect that their "knowledge" comes from old information, published to straighten out the problems that came from incorrectly calling CUP PSI.

If two variables are correlated, you can estimate one from the other. The opposite of this is "statistically independent", which means that you can't estimate one from another. Actually, it is very hard to come up with numbers that are completely statistically independent, or uncorrelated. Usually the question is not whether things are correlated, but how well they are correlated. If you plot my weight vs. my belt size for the past 20 years (please don't!), you'll find that one variable reasonably predicts the other. My belt size and weight, then, are correlated. They won't be perfectly correlated, and they might not be linearly correlated, but they will be well correlated.

[1] Lyman 47th Reloading Handbook, p92

A figure of merit for correlation is the R^2 value. In the simple case of linear regression, an R^2 of .8 means that 80% of the variation in one variable is "controlled" by the other, and the remaining 20% of the variation is unaccounted for. Run regression on a pair of columns of random numbers, and you'll get R^2 values from a fraction of a percent to a few percent. Run it on a very precise micrometer's reading vs. the marked values on a set of gauge blocks spanning a couple of inches, and you'll get something very close to 100%.

It's a fact that two variables that are both well correlated with a third variable must be well correlated to each other. So if the copper crusher system is well correlated with peak chamber pressure, and the piezoelectric PSI system is well correlated with peak chamber pressure, then CUP must be well correlated with piezoelectric PSI. It cannot be otherwise.

All measurement systems lie, at least a little bit. Like all measurement systems, the CUP method and the PSI method both have a certain amount of random error in them. From published data, (Lyman manual, p91), it is easy to estimate the random error in both systems. The bottom line is that the random error associated with the CUP system has a standard deviation of about 2,000 PSI (correct usage), and the piezoelectric system has a standard deviation of about 1,300 PSI.

This random variation in the measurement systems accounts for part of the puzzlement in attempting the conversion. The 7x57 Mauser is rated 46,000 CUP, and 51,000 PSI. The 300 Savage is also rated 46,000 CUP, but 47,000 PSI. Random error in both measurement systems accounts for this discrepancy. Because there is random error in both measurement systems, any conversion will be approximate, rather than absolutely precise.

While it is true that the deformation of the copper pellet in the crusher system is influenced by all the pressure that happens during the discharge of a bullet, it is also true that the main thing that the CUP system measures is peak chamber pressure. The deformation that happens "off-peak" is properly regarded as measurement system error, and it is minimal, as I will show a bit down the page.

Searching for Correlation Between PSI and CUP

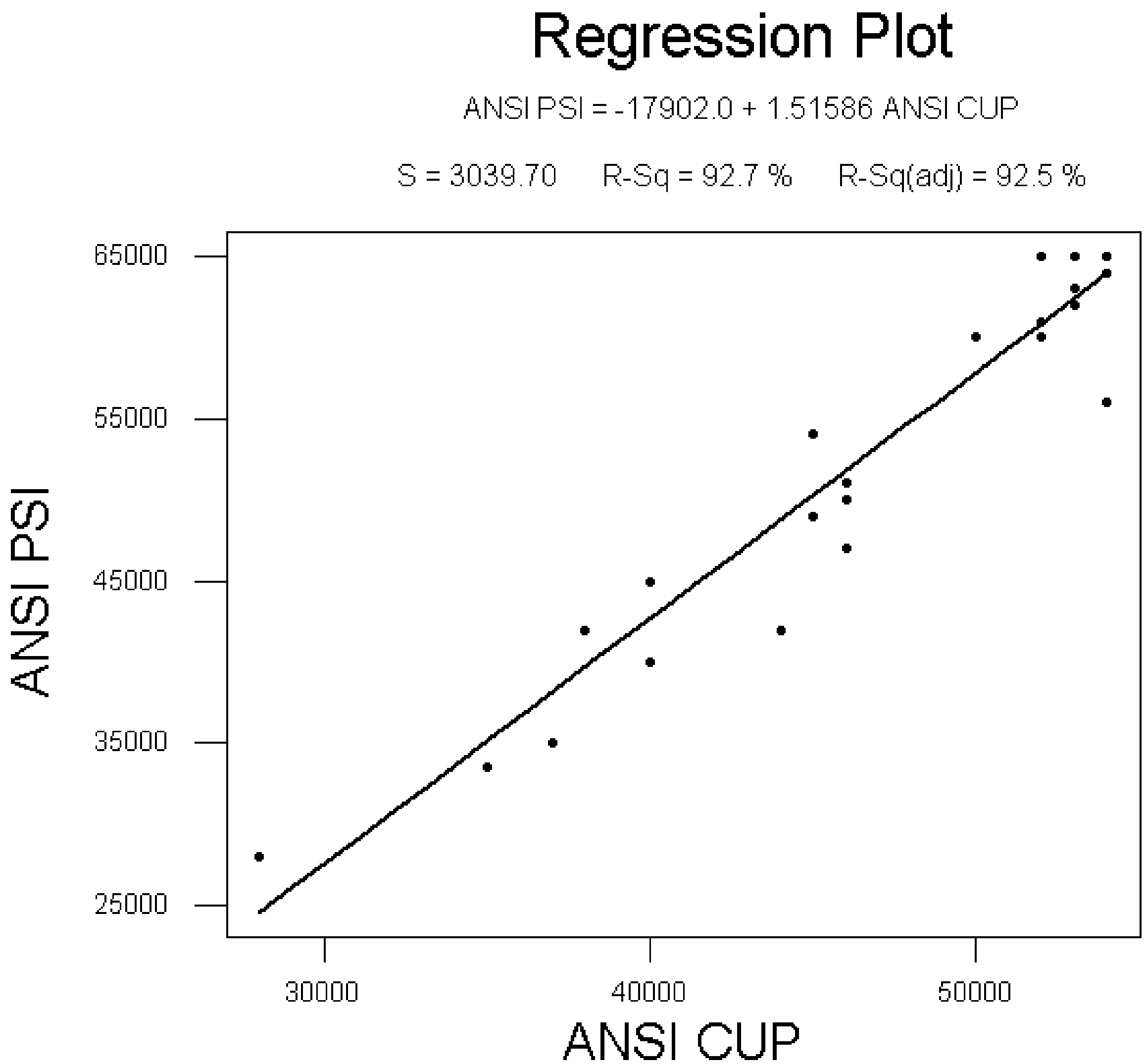
Testing for this correlation is easy. All we need is a set of measurements where the same event was measured in both systems, and we need that set of measurements to span a large enough range that we can "see" the correlation above the random error that is present.

Measurements taken simultaneously on several examples of a single handload would about the worst possible choice of data sets. Careful handloaders try very hard to minimize variation. Ideally, the pressure variation from cartridge to cartridge is zero. In practice, the range of pressures is so small that a regression on that data would be completely swamped by random measurement error, which is significant in both the CUP and piezoelectric systems.

There is a much better alternative. There are cases where SAAMI has set maximum pressures for rifles in both CUP and PSI. That data set spans a few tens of thousands of PSI, and, assuming that SAAMI was careful in how they set the limits, it is much better for our purpose. I have access to about 30 such data pairs, and that is enough to provide a reasonable estimate of the conversion factor.

Cartridge	**ANSI CUP**	**ANSI PSI**
222 rem	46000	50000
22-250 rem	53000	65000
243 win	52000	60000
25-06 rem	53000	63000
257 roberts	45000	54000
264 win mag	54000	64000
270 win	52000	65000
280 rem	50000	60000
284 win	54000	56000
30 carbine	40000	40000
300 savage	46000	47000
300 win mag	54000	64000
30-06 springfield	50000	60000
303 british	45000	49000
30-30 win	38000	42000
308 win	52000	60000
32 win special	38000	42000
338 win mag	54000	64000
35 rem	35000	33500
375 h&h mag	53000	62000
444 marlin	44000	42000
45-70 government	28000	28000
6.5 rem mag	53000	65000
6mm rem	52000	65000
7mm express Rem	40000	45000
7mm rem mag	46000	51000
7mm SE vH	52000	61000
7x50 R	52000	61000
8mm rem mag	37000	35000
8x50R	54000	65000

Submitting the SAAMI/ANSI numbers to regression, we get this:



Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	3.302E+09	3.302E+09	357.419	0.000
Error	28	258713297	9239761		
Total	29	3.561E+09			

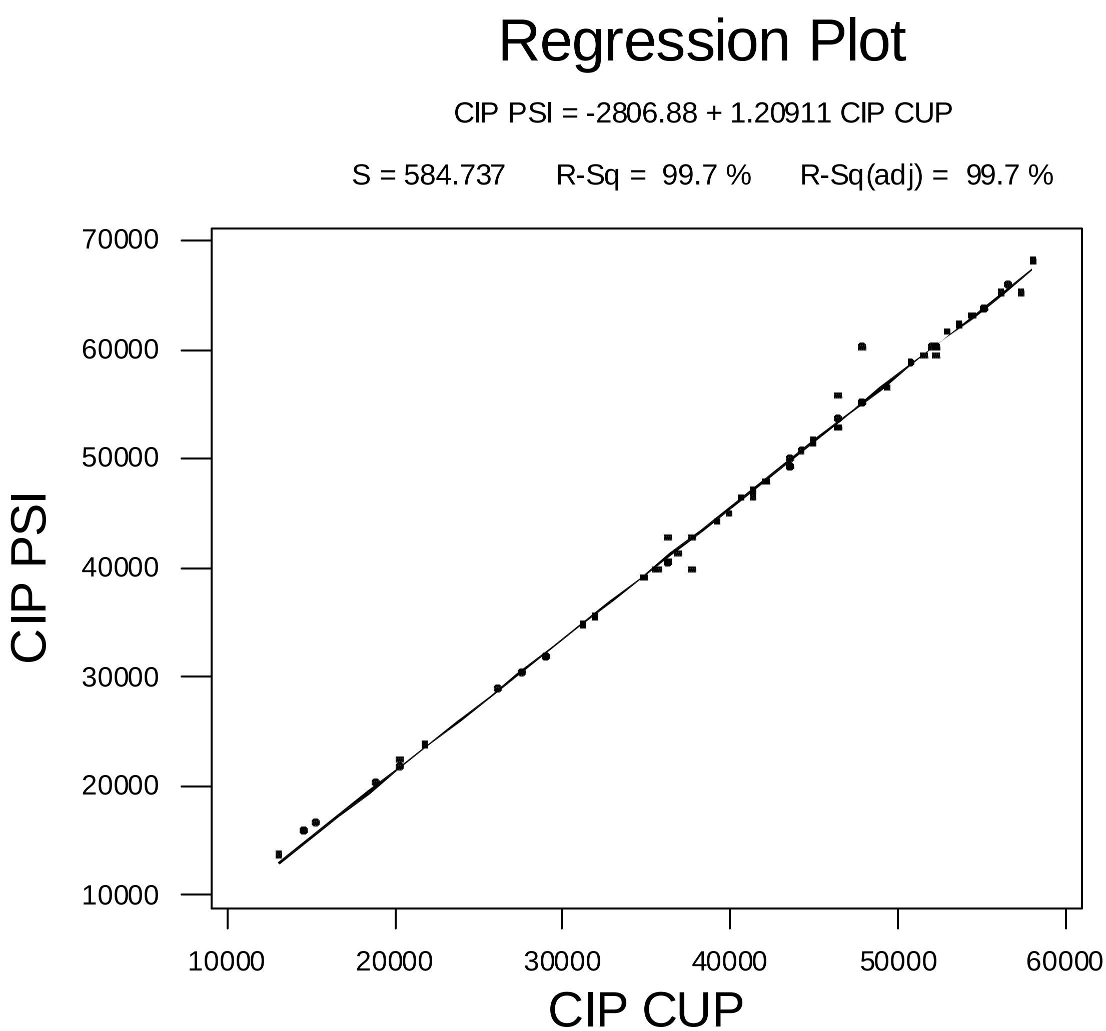
An R^2 value of .927 puts an end to all discussion about whether PSI and CUP are correlated. They are. To prove otherwise, you would have to prove that .927 is a lot closer to zero than it is to one, and you'd have to show that the data pattern in the graph is much more like a shotgun pattern than it is like a straight line. An F value in the low teens is usually enough to show statistical significance, and we have an F value of 357.4.

If two variables are well correlated, there is always a formula for converting from one to the other. The formula for converting from CUP to PSI is shown at the top of the graph. Since the numbers you are converting do not precisely represent actual chamber pressure, the results you get from the conversion will not be precise. About 2/3 of the time, the formula will land you within 3,000 PSI, so exercise appropriate caution. Also, do not attempt to use this conversion for handguns or shotguns, or to use it outside the range shown. We don't yet know how the conversion works outside the data we have studied.

Let's go through a couple of examples to show how the formula works. The formula PSI = -17,902 + 1.516 x CUP is useful if you have data published in CUP, and want to compare with data published in PSI, Or, if you're like me, and have instrumented one or more rifles with strain gauges, you might want to use published CUP data to set an approximate limit for your loads in PSI. My lovely 6.5x55 Swede is rated at 46,000 CUP, and has no PSI rating. What should I use for a limit in PSI? Multiplying 46,000 by 1.516, and subtracting 17,902 gives me an upper limit of about 51,834 PSI. If I graduate to a .416 Rigby, which is rated at 42,000 CUP, the same calculation gives us 45,770 PSI. Reversing the math the 7mm Weatherby Magnum is rated at 65,000 PSI, with no corresponding CUP number. Converting 65,000 PSI results in a stout 54,685 CUP.

For reasons unknown to me, the 223 Rem doesn't appear in either of the data sets I have access to. It is also statistically very different from the rest of the data.

There is also a separate European CIP standard, which uses a different procedure, and produces different results. Data for 191 cartridges is readily available. Their curve and



formula look like this:

The European CIP conversion is much more precise than the US SAAMI conversion. If you eliminate the statistically peculiar 280 FI NE, 310 Cadet Rifle, 38-40 Win, 44-40 Win, 7x50 R, 7x75 R SE vH, 8mm Rem Mag, and the 32 Rem, all other conversions from CUP to PSI are within about 850 PSI. The precision of the conversion, and the fact that the same exact values pop up again and again in the residuals indicates that the Europeans have probably actually just been using one system, and converting by linear formula to produce the second set of numbers.

Conclusions

1. PSI (correct use) is highly correlated to CUP. Evidence: R^2 = .927 makes it impossible to successfully argue otherwise.

2. CUP is mainly an indicator of peak chamber pressure: Evidence: The way that piezoelectric systems are commonly used, they report purely peak chamber pressure. The CUP system is highly correlated with the piezoelectric system. If the “off-peak” deformation of the copper pellet were large, the correlation to the piezoelectric system would be poor.

3. SAAMI did a pretty consistent job of setting maximum pressure limits in both systems. Evidence: The two are highly correlated. Basically, they got pretty close to the same answer both ways.

4. You can convert from one system of measurement to the other. Evidence: Definition of "correlated". Basically, correlated means that you can estimate one variable from the other. The opposite of this is "statistically independent", which means that you can't.

5. The formula for the conversion is the one shown above. Evidence: Produces the "least squares fit" for the two systems, and it produces an R^2 of .927. You can test the formula by plugging in any of the CUP numbers shown above. The formula will give you back a PSI number that is close to the one shown in the table.

6. Work remains to be done in refining the SAAMI conversion. Evidence: An R^2 of 92.7% is produced, leaving 7.3% of the variation to be explained. Measurement system error probably sets the limit of the R^2 that can be obtained at around 98%. That leaves 5.3% of the variation unexplained. Perhaps someone can discover what the unaccounted for variable is.

7. The first example of something disproves all claims that it does not exist. The formula exists, and it works. So all claims that it does not exist cannot be true.